Discrete Time Processing of Continuous time Signals

Continuous Time Processing of Discrete Time Signals

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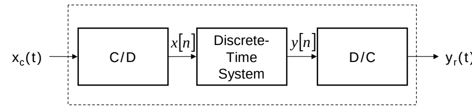
Discrete-Time Processing of Continuous-Time Signals



- Overall system is equivalent to a continuous-time system
  - Input and output is continuous-time
- The continuous-time system depends on
  - Discrete-time system
  - Sampling rate
- We're interested in the equivalent frequency response
  - First step is the relation between $x_c(t)$ and x[n]
  - Next between y[n] and x[n]
  - Finally between $y_r(t)$ and y[n]

LTI Discrete Time System - Effective Frequency Response

- Input continuous-time to discrete-time

$$x[n] = x_c(nT) \qquad X\left(e^{j\omega}\right) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

- Assume a discrete-time LTI system

$$Y\left(e^{j\omega}\right) = H\left(e^{j\omega}\right)X\left(e^{j\omega}\right) = \frac{1}{T}H\left(e^{j\omega}\right)\sum_{k=-\infty}^{\infty} X_c\left(j\left(\frac{\omega}{T} - \frac{2\pi k}{T}\right)\right)$$

- Output discrete-time to continuous-time

$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n]\frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \qquad Y_r(j\Omega) = \begin{cases} TY\left(e^{j\Omega T}\right) & |\Omega| < \pi/T \\ 0 & \text{otherwise} \end{cases}$$

- Output frequency response

$$Y_r(j\Omega) = \begin{cases} H\left(e^{j\Omega T}\right)X_c(j\Omega) & |\Omega| < \pi/T \\ 0 & \text{otherwise} \end{cases}$$

- Effective Frequency Response

$$Y_r(j\Omega) = H_{eff}(j\Omega)X_c(j\Omega) \qquad H_{eff}(j\Omega) = \begin{cases} H\left(e^{j\Omega T}\right) & |\Omega| < \pi/T \\ 0 & \text{otherwise} \end{cases}$$

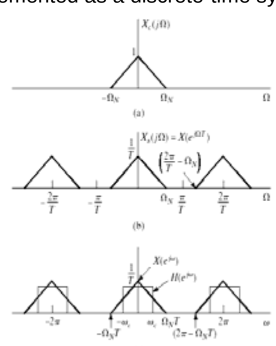
Example

- Ideal low-pass filter implemented as a discrete-time system

Continuous-time Input signal

Sampled continuous-time Input signal

Apply discrete-time LPF

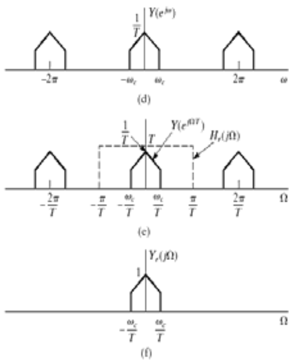


Example Continued

Signal after discrete-time LPF is applied

Application of reconstruction filter

Output continuous-time signal after reconstruction



Impulse Invariance

- Given a continuous time system $H_c(j\Omega)$
  - how to choose discrete time system response $H(e^{j\omega})$
  - so that effective response of discrete time system $H_{eff}(j\Omega)=H_c(j\Omega)$
- Answer:

$$H(e^{j\omega}) = H_c(j\omega/T) \qquad |\omega| < \pi$$

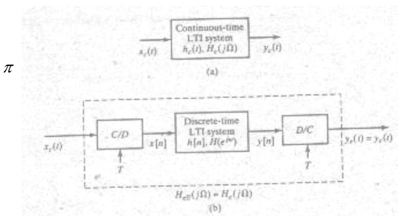


Figure 4.15 (a) Continuous-time LTI system. (b) Equivalent system for bandlimited inputs.

Impulse Invariance

- Condition:
- Given these conditions the discrete time impulse response can be written in terms of continuous time impulse response as

$$H_c(j\Omega) = 0 \qquad |\Omega| \geq \pi/T$$

- Resulting Discrete Time system is the impulse invariant version of the continuous time system

$$h[n] = Th_c(nT)$$

Example: Discrete Time Low-Pass Filter By Impulse Invariance

- Ideal low pass discrete time filter by impulse invariance

$$H_c(j\Omega) = \begin{cases} 1 & |\Omega| < \Omega_c \\ 0 & else \end{cases}$$

- The impulse response of continuous time system is

$$h_c(t) = \frac{\sin(\Omega_c t)}{\pi t}$$

- Obtain discrete time impulse response via impulse invariance

$$h[n] = Th_c(nT) = T\frac{\sin(\Omega_c nT)}{\pi nT} = \frac{\sin(\omega_c n)}{\pi n}$$

- The frequency response of the discrete time system is

$$H_c(e^{j\omega}) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \omega_c < |\omega| \leq \pi \end{cases}$$

Continuous time Processing of Discrete Time Signals

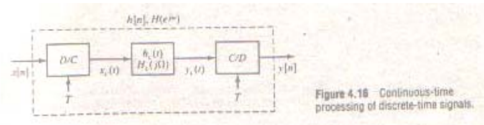


Figure 4.16 Continuous-time processing of discrete-time signals.

$$x_c(t) = \sum_{n=-\infty}^{\infty} x[n] \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \qquad X_c(j\Omega) = TX\left(e^{j\Omega T}\right), \quad |\Omega| < \pi/T$$

$$y_c(t) = \sum_{n=-\infty}^{\infty} x_c(t) \frac{\sin[\pi(t-nT)/T]}{\pi(t-nT)/T} \qquad Y_c(j\Omega) = X_c(j\Omega)H_c(j\Omega), \quad |\Omega| < \pi/T$$

$$Y\left(e^{j\omega}\right) = \frac{1}{T} Y_c\left(j\frac{\omega}{T}\right), \quad |\omega| < \pi$$

$$H\left(e^{j\omega}\right) = H_c\left(j\frac{\omega}{T}\right), \quad |\omega| < \pi$$

Example : Non- Integer Delay

$$H\left(e^{j\omega}\right) = e^{-j\omega\Delta}, \qquad |\omega| < \pi$$

$$y[n] = y_c(nT) = x_c[nT - \Delta T]$$

$$y[n] = \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\pi\left(t - \Delta T - kT\right)/T\right]}{\pi\left(t - \Delta T - kT\right)/T}\Bigg|_{t=nT}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \frac{\sin\left[\pi\left(n - k - \Delta\right)\right]}{\pi\left(n - k - \Delta\right)}$$

$$h[n] = \frac{\sin \pi(n-\Delta)}{\pi(n-\Delta)}, \qquad -\infty < n < \infty$$